Robust Distributed Compression with Learned Heegard—Berger Scheme

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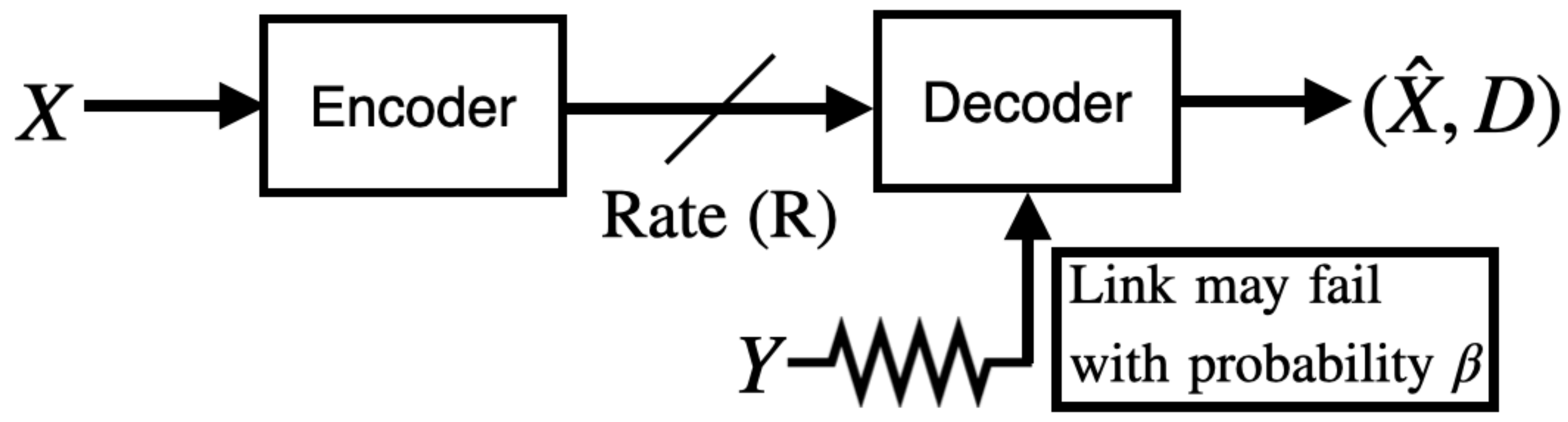
I - Overview

- Distributed compression can improve transmission rates in practical applications such as sensor networks and federated learning by leveraging side information, compared to point-to-point compression.



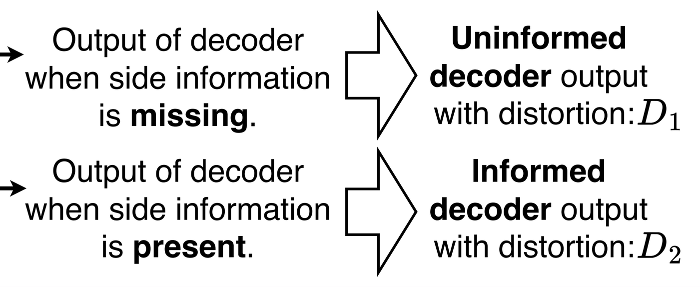
- What happens if the side information fails to reach to the decoder? **What if the side information may be absent?**
  - ➢ Heegard—Berger (1985) gave theoretical asymptotic limits, but constructive codes are missing in the literature.

The R-D function for X with side information Y may be missing is:

$$R(\mathcal{D}) = \min_{p(w,u|x)} \left(I(X;W) + I(X;U|Y,W)\right),$$

under weighted distortion constraint

$$\mathbb{E}[\beta d(X, \hat{X}_1) + (1-\beta) d(X, \hat{X}_2)] \leq \mathcal{D}.$$



Achievability relies on **random binning** arguments.

- The universal approximation capability of neural networks enables the design of **one-shot distributed neural compressors** for real-life applications. However, ensuring the robustness of neural compressors against link failures is essential for their practical deployments.

In our work, we

- ➢ **propose a single-shot learned neural compressor for the Heegard—Berger setup,**
- ➢ **demonstrate the grouping behaviour of compressors akin to the 'random binning' arguments of asymptotic results.**

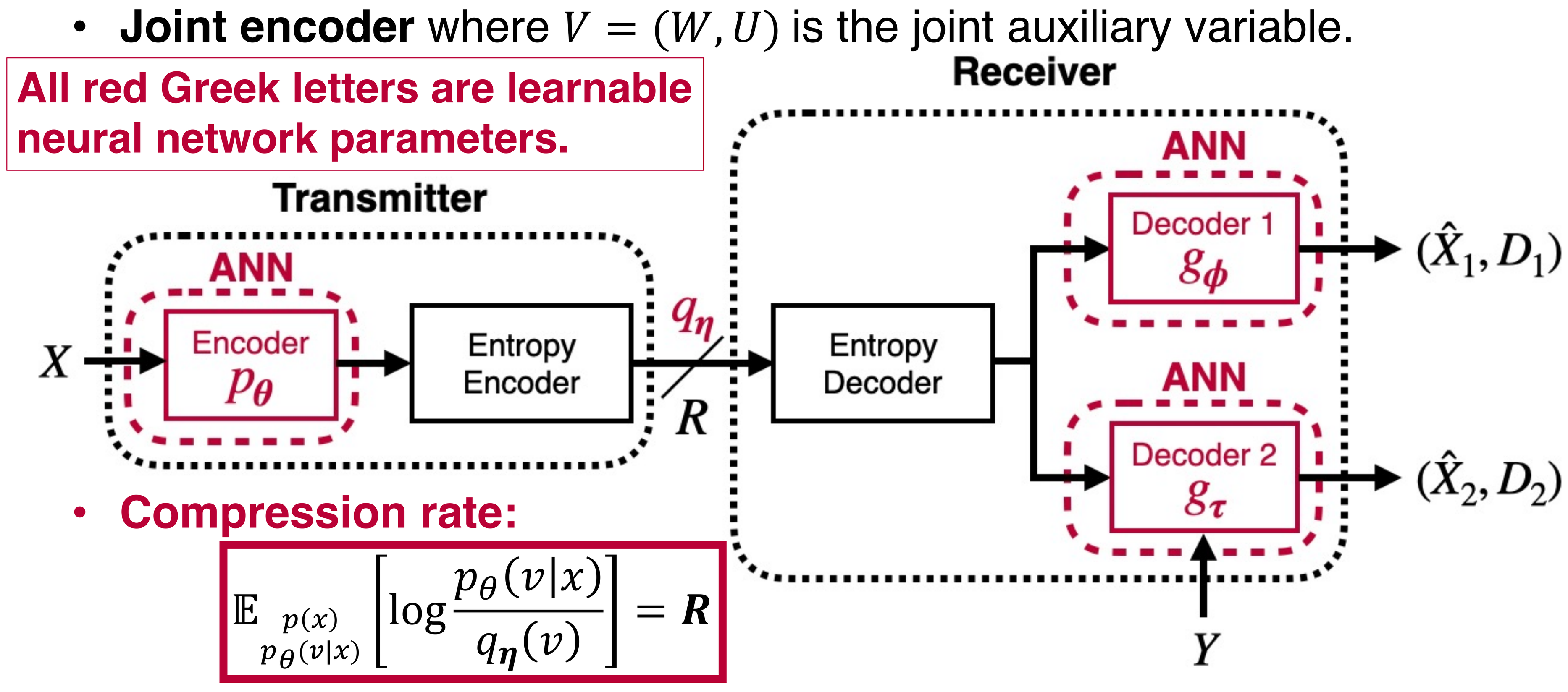
II – Operational Neural Schemes

We present three unique constructive solutions to Heegard—Berger problem expanding on the work from Ozyilkan et al. 2023.

1) Joint Model

- **Joint encoder** where $V = (W, U)$ is the joint auxiliary variable.

All red Greek letters are learnable neural network parameters.

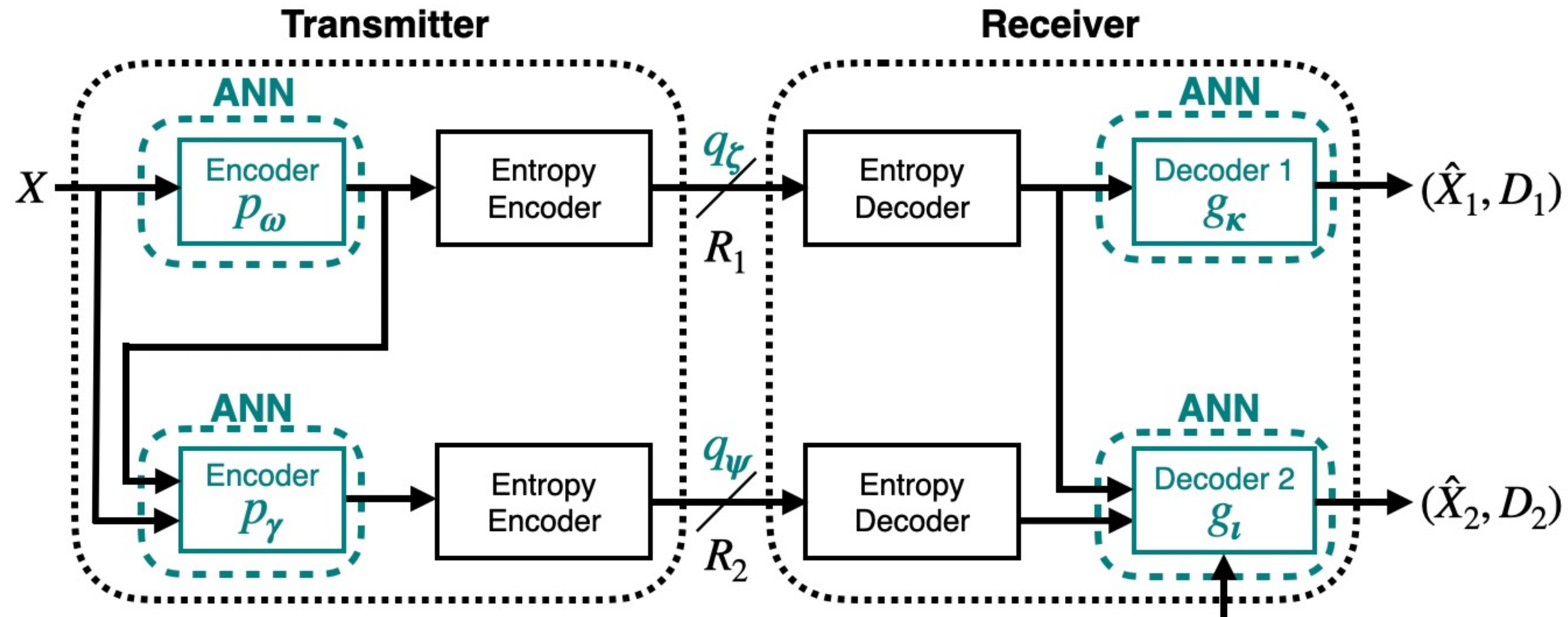


- **Compression rate:**

$$\mathbb{E}_{\substack{p(x)\\ p_\theta(v|x)}}\left[\log \frac{p_\theta(v|x)}{q_\eta(v)}\right] = R$$

2) Marginal Model

- **Separate encoders** where W and U are separate auxiliary variables.



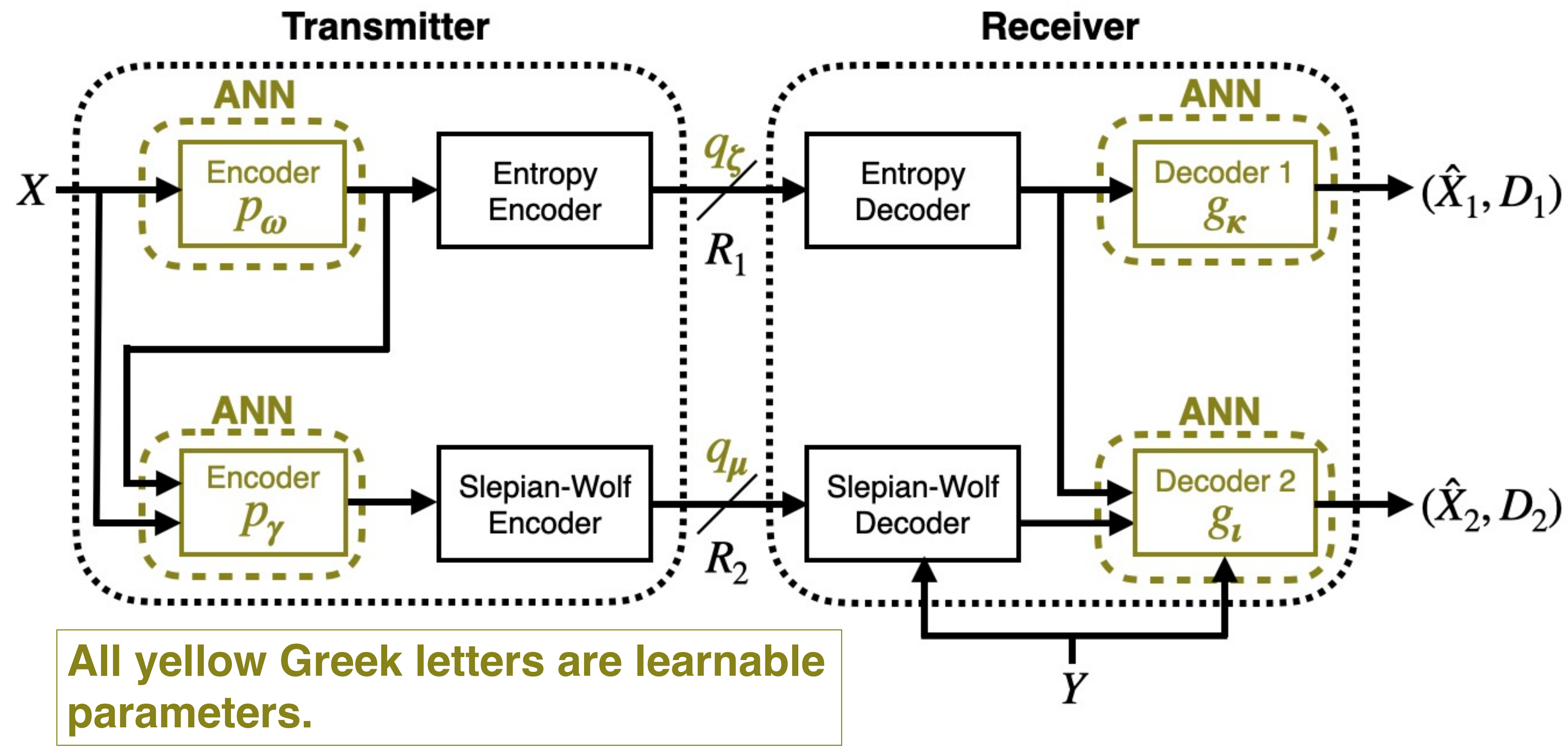
All blue Greek letters are learnable parameters.

- **Compression Rate:**

$$\mathbb{E}_{\substack{p(x)\\ p_\omega(w|x)\\ p_\gamma(u|w,x)}}\left[\log \frac{p_\omega(w|x)}{q_\zeta(w)} + \log \frac{p_\gamma(u|w,x)}{q_\psi(u|w)}\right] = \underbrace{R}_{R_1 + R_2}$$

3) Conditional Model

- **Separate encoders** where W and U are separate auxiliary variables. Moreover, entropy model is **conditioned on side information**.



All yellow Greek letters are learnable parameters.

- **Compression Rate:**

$$\mathbb{E}_{\substack{p(x)\\ p_\omega(w|x)\\ p_\gamma(u|w,x)}}\left[\log \frac{p_\omega(w|x)}{q_\zeta(w)} + \log \frac{p_\gamma(u|w,x)}{q_\mu(u|w,y)}\right] = \underbrace{R}_{R_1 + R_2}$$

For All Three Models:

- The encoder outputs are discrete.
- There is one **informed** and one **uninformed** decoder. The weighted **distortion**:

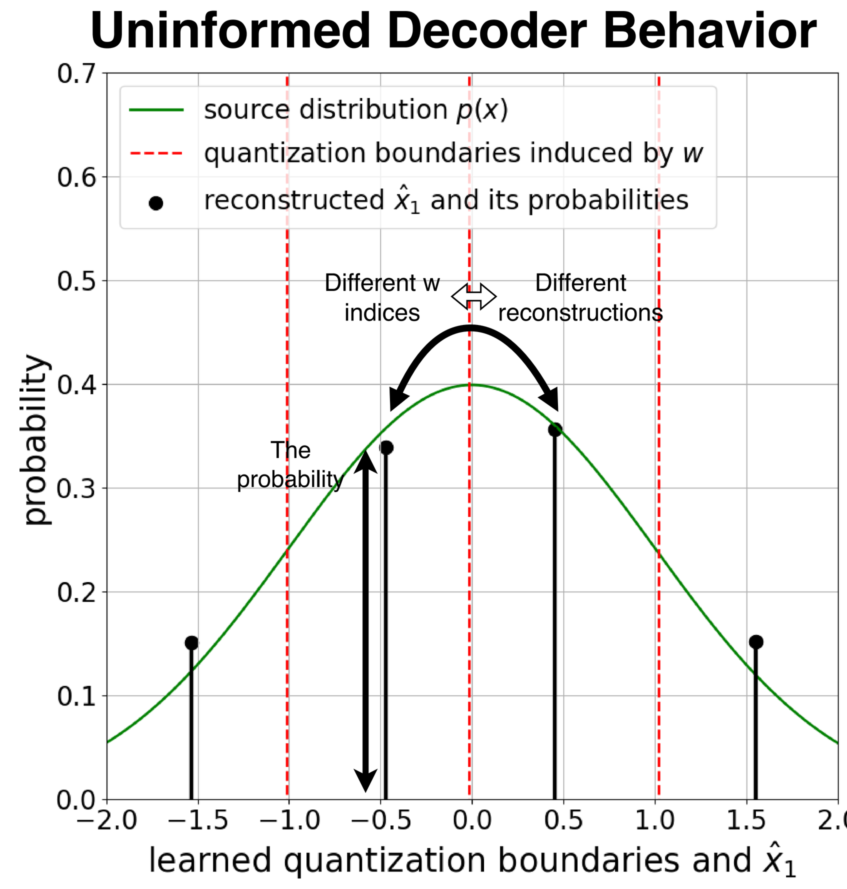
$$\mathcal{D} = \mathbb{E}_{p(x)}[\beta d(X, \hat{X}_1) + (1-\beta)\beta d(X, \hat{X}_2)]$$

- **Lagrangian** as training loss: $\mathfrak{L} = \boldsymbol{R} + \lambda \cdot \boldsymbol{\mathcal{D}}$

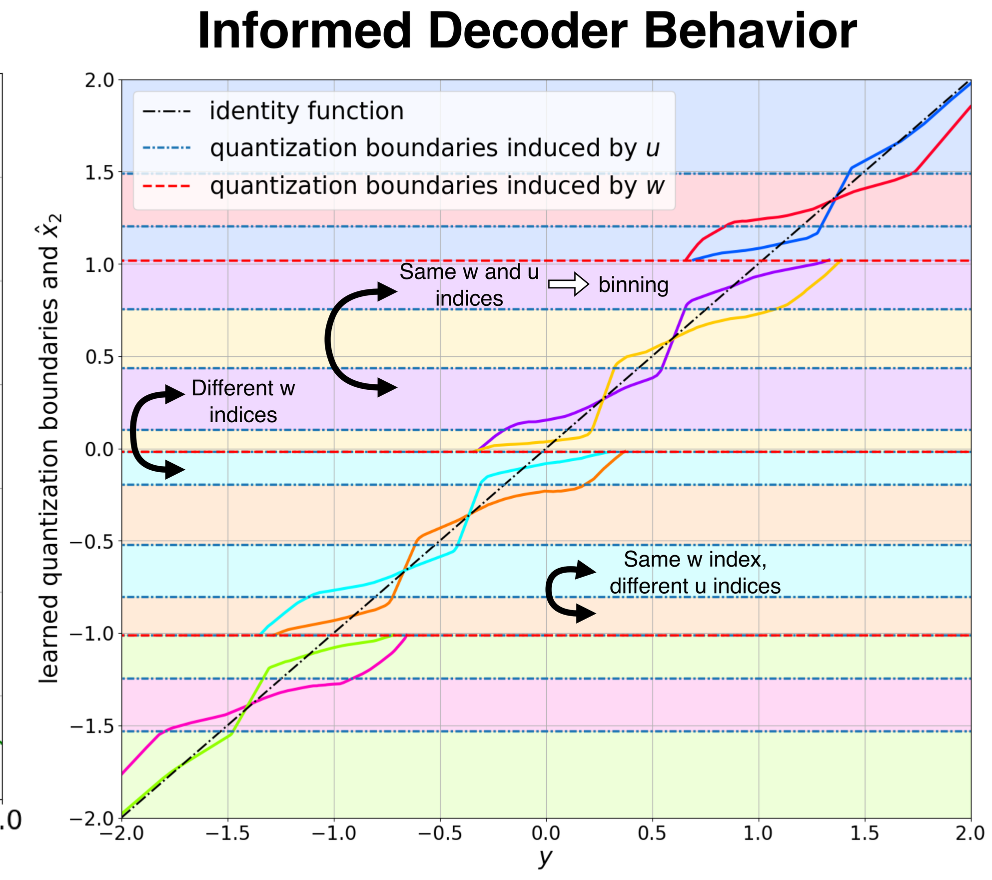
III - Neural Compressors are Robust

- We choose the X and Y to be i.i.d. Gaussian sources with quadratic distortion measure since their R-D curves are well-studied, allowing us to assess how close we get to R-D curves. For $Y = X + N$ with $X \sim N(0,1)$ and $X \sim N(0, 10^{-2})$ and $\beta = 0.2$, marginal model behaviour is as follows:

Uninformed Decoder Behavior

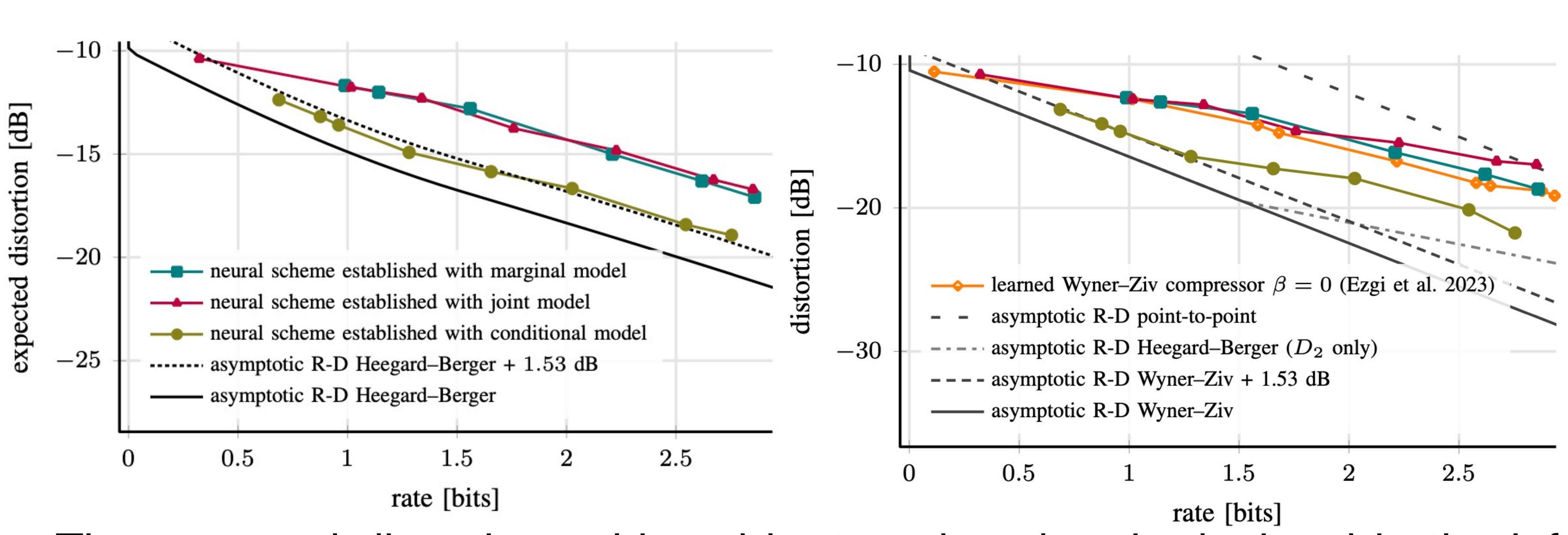


Informed Decoder Behavior



The colors between each boundary represent a unique (w, u) pair. Solid lines is the output of the decoding function, each representing a different pair of (w, u) as inputs within its respective quantization region. **Grouping behavior is evident.**

- We compare the R-D performances with theoretical limits. For $Y = X + N$ with $X \sim N(0,1)$ and $X \sim N(0, 10^{-1})$ and $\beta = 0.01$, R-D curves are:



The expected distortion achieved by two decoders is depicted in the left panel, while the distortion attained by the informed decoder alone is depicted in the right panel. Right panel shows **the trade-off between system robustness and compression efficiency.**